Forum for Academic Discussion on Development of Distance Function Preserving Finite Element Level Set Methods

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The special Project Fund Grant from AC21 has assisted in developing research links between three AC21 partners: Shanghai Jiao Tong University, Jilin University and North Carolina State University. Participants from AC21 partner universities and other non-AC21 institutions will be able to exchange information about the latest advances in their research areas, as well as plan future initiatives based on contacts made during their collaboration.

Our three universities have worked together to establish an interdisciplinary team, and held an international forum on Mathematics to discuss new directions in our research field. The forum was held in Shanghai Jiao Tong University from Jan 10-12, 2014. Besides four representatives of the three AC 21 partners, with the kind help of other international cooperative universities we were able to invite top-level researchers from each specialist field to participate as invited speakers (17 persons): from the University of Oxford, University of Pierre-and-Marie-Curie (Paris VI), University of Wisconsin at Madison, Pennsylvania State University, University of Konstanz, University of New South Wales, Hong Kong University of Science and Technology, Chinese University of Hong Kong, and City University of Hong Kong. The project will help to build an international research team, set up long-term cooperative relationships to provide a platform and mechanism for future academic exchanges, involve other international institutions to attract research and project funding from national and international organizations, and promote student exchange for postgraduate and research training in the AC21 network and other institutions.

The summaries of our group's major works this year are listed briefly below:

(1) A new realization of a finite element level set method for simulation of immiscible fluid flows was introduced and validated on numerical benchmarks. The new method involves a mixed discretization of the dependent variables, discretizing the flow variables with non-conforming Rannacher-Turek finite elements while using a simple first order conforming discretization of the level set field. A three-step

egregated solution approach is employed: first, a discrete projection method is used to de-couple and compute the velocity and pressure separately, after which the level set field can be computed independently. The developed method was tested and validated using a static bubble test case and on a numerical rising bubble test case, for which a very accurate benchmark solution has been established. The new approach was also compared against two commercial simulation codes, Ansys Fluent and Comsol Multiphysics, which showed that the method we developed is a magnitude or more accurate and at the same time significantly faster than state-of-the-art commercial codes.

(2) The performance of two commercial simulation codes, Ansys Fluent and Comsol Multiphysics, was thoroughly examined for a recently established two-phase flow benchmark test case. In addition, the commercial codes were directly compared with the newly developed academic code, FeatFlow TP2D. The results of this study show that the commercial codes fail to converge and produce accurate results, and leave much to be desired with respect to direct numerical simulation of lows with free interfaces. The academic code, on the other hand, was shown to be computationally efficient, produced very accurate results, and outperformed the commercial codes by a magnitude or more.

(3) Numerical simulation of incompressible multiphase flows with immiscible fluids is still a challenging field, particularly for 3D configurations undergoing complex topological changes. In this paper, we discussed a 3D FEMapproach with a high-order Stokes elements (Q2/P1) for velocity and pressure on general hexahedral meshes. A discontinuous Galerkin approach with piecewise linear polynomials (dG(1)) was used to treat the level set function. The developed methodology allows the application of special redistancing algorithms which do not change the position of the interface. We explained the corresponding FEM techniques for treating the advection steps and surface tension effects, and validated the corresponding 3D code with respect to both numerical test cases and experimental data. The corresponding applications describe the classical rising bubble problem for various parameters and the generation of droplets from a viscous liquid jet in acoflowing surrounding fluid. Both of these applications can be used for rigorous benchmarking of 3D multiphase flow simulations.

To sum up, we have co-published one SCI paper in a top research journal, *Journal of Computational Physics*, successfully held the "SJTU International Forum on Mathematics" in January 2014, developed close collaboration with international research teams, and promoted student exchange for postgraduates and research training within the AC21 network. We have realized most of the aims and objectives of the project.

In addition to the development of the research field on the distance function preserving finite element level set methods, there are a number of sub-themes that can be embraced under this International Forum theme, such as PDEs, Numerics, Stochastics, Financial Mathematics, and Applied Mathematics, and 17 top-level speakers from 9 world-class universities delivered presentations in the forum to exchange ideas about both academic topics and international cooperative programs; therefore, we have changed our forum's name to "SJTU International Forum on Mathematics", which we feel will be more suitable for this forum.

Lastly, we are pleased to offer our best thanks to the AC21 Special Project Fund and AC21 General Secretariat.

The website of the forum as below: